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STRESS CONCENTRATIONS IN FILAMENT-STIFFENED SHEETS OF FINITE LENGTH

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16. Abstract

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SUMMARY

A simple model of filamentary composite material is employed to investigate stress concentrations in a filament-stiffened sheet of finite length. The model is composed of a single layer of parallel, tension-carrying filaments embedded in a shear-carrying matrix. The sheet is of finite length in the filament direction and of infinite length normal to the filament direction. Filament stress-concentration factors are calculated as functions of the number of broken filaments and a length-stiffness parameter for the cases of uniform normal edge load and uniform normal edge displacement. In the uniform-edge-load case, the stress-concentration factors are found to increase with decreasing filament length. The opposite effect is noted in the uniform-edge-displacement case where, in addition, the stress-concentration factor is found to have an upper limit which is fixed by the value of the length-stiffness parameter.

INTRODUCTION

In the design and application of aerospace structures the problem of stress concentration is one of continuing importance. In recent years conventional engineering metals have been replaced in some weight-critical applications by filamentary composite materials. These materials behave somewhat differently from conventional metals when weakened by flaws and, hence, different analytical models must be used to investigate their behavior.

One simplified model for filamentary-composite-material behavior which has been found useful is the "stringer-sheet" model employed in reference 1 for investigating static and dynamic stress concentrations around broken filaments. This model consists of a sheet of parallel, equally spaced, tension-carrying filaments embedded in a shear-carrying matrix. In references 2 and 3 the model has been used to investigate the interaction of neighboring groups of broken filaments, the matrix shear forces, and the recovery of load by broken filaments. In reference 4 a similar model has been employed to study stress concentration in two-dimensional arrays of equally spaced filaments.

In all of these studies attention has been restricted to bodies of infinite extent. It is well known, however, that results for infinite bodies cannot always be applied indiscriminately to similar problems involving finite specimens. It is desirable, therefore, to assess the influence of specimen size on stress concentration. In the present paper the stringer-sheet model is employed to investigate static stress concentration due to broken filaments in a filament-stiffened sheet of infinite length normal to the filament direction and of finite length in the filament direction. Stress-concentration factors are found for a central cut across various numbers of adjacent filaments and for a wide range of values of a nondimensional length-stiffness parameter.

SYMBOLS

d	filament spacing
EA	extensional stiffness of a filament
Gh	matrix shear stiffness per unit length
K_r	stress-concentration factor for r broken filaments
k	length-stiffness parameter, $l\sqrt{\frac{Gh}{EAd}}$
l	half-length of filament sheet
$\overline{M}$	transformed influence functions
M_n	displacement of nth filament for influence-function solution
m,n,r	indexes
N_n	load in nth filament for influence-function solution
P_n	dimensionless load in nth filament, p_n/p
p	filament load parameter
p_n	load in nth filament
r	number of broken filaments

U_n dimensionless displacement of nth filament, $\frac{EA}{pl} u_n$

u_n displacement of nth filament

x coordinate parallel to filament

η dimensionless coordinate parallel to filaments, $\frac{x}{l}$

θ transform variable

Superscript:

∞ denotes values for infinite-length sheet

ANALYSIS

The finite-length sheet under investigation is shown in figure 1 along with the coordinate system and some notation. The filaments are separated by a constant distance d and are numbered from $-\infty$ to ∞ from left to right. The coordinate along the filaments is x and the displacement of the nth filament at location x is denoted by $u_n(x)$. The force in the nth filament, taken to be positive in tension, is $p_n(x)$ and is given in terms of u_n by

$$p_n = EA \frac{du_n}{dx} \tag{1}$$

where EA is the extensional stiffness of the filament. The matrix shear force per unit length between the nth and (n + 1)th filaments is given by $\frac{Gh}{d}\left(u_{n+1} - u_n\right)$, where Gh is the shear stiffness of the matrix. Static equilibrium of an element of the nth filament requires that

$$EA \frac{d^2u_n}{dx^2} + \frac{Gh}{d}\left(u_{n+1} - 2u_n + u_{n-1}\right) = 0 \tag{2}$$

In figure 1 filaments 0 and 1 are shown broken at $x = 0$. In general, for r broken filaments, let $0 \leqq n \leqq r - 1$ denote the broken filaments. Then, the appropriate boundary conditions along the line containing the cut $(x = 0)$ are

$$\left.\begin{array}{ll} p_n(0) = 0 & (0 \leqq n \leqq r - 1) \\ u_n(0) = 0 & (n < 0 \quad \text{or} \quad n \geqq r) \end{array}\right\} \quad . \tag{3}$$

Two sets of boundary conditions at the edges of the filamentary sheet are of interest. These are as follows: (I) uniform normal displacement, and (II) uniform normal load. In terms of the pertinent variables, the two conditions are

$$\text{Problem I:} \quad u_n(\pm l) = \pm \frac{pl}{EA} \tag{4}$$

and

$$\text{Problem II:} \quad p_n(\pm l) = p \tag{5}$$

(Note that these two conditions would be equivalent if no filaments were broken.) Problems I and II are defined by equations (2) and (3) along with equations (4) and (5), respectively.

Nondimensionalization

In order to cast the problems in a more convenient form, let

$$\left.\begin{array}{l} p_n = pP_n \\ u_n = \frac{pl}{EA} U_n \\ x = l\eta \end{array}\right\} \tag{6}$$

Then, equation (2) becomes

$$\frac{d^2U_n}{d\eta^2} + k^2 \left(U_{n+1} - 2U_n + U_{n-1}\right) = 0 \tag{7}$$

where k is a length-stiffness parameter defined by $l\sqrt{\frac{Gh}{EAd}}$. For the two problems considered here the complete sets of boundary conditions become

$$\left.\begin{array}{ll} P_n(0) = 0 & (0 \leqq n \leqq r - 1) \\ U_n(0) = 0 & (n < 0 \quad \text{or} \quad n \geqq r) \end{array}\right\} \tag{8}$$

and

$$\text{Problem I:} \quad U_n(\pm 1) = \pm 1 \tag{9a}$$

and

$$\text{Problem II:} \quad P_n(\pm 1) = 1 \tag{9b}$$

The dimensionless forces and displacements are related by

$$P_n(\eta) = \frac{dU_n}{d\eta}(\eta) \tag{10}$$

Solution of problems I and II is complicated by the fact that the boundary conditions at $x = 0$ are mixed; that is, load is prescribed over one part of the boundary, with displacement prescribed over the remainder. To overcome this difficulty the influence-function technique introduced in reference 1, and later used in references 2 and 3, is again employed. It is assumed that the force and displacement in the nth filament have the form

$$P_n(\eta) = 1 + \sum_{m=-\infty}^{\infty} N_{n-m}(\eta) U_m(0)$$

and

$$U_n(\eta) = \eta + \sum_{m=-\infty}^{\infty} M_{n-m}(\eta) U_m(0)$$

where

$$\frac{dM_n(\eta)}{d\eta} = N_n(\eta)$$

The quantities M_n and N_n are, respectively, the displacement and load in the nth filament due to a unit displacement of the zeroeth filament at $\eta = 0$. Application of the boundary conditions at $\eta = 0$ (eq. (8)) yields

$$\left.\begin{aligned} P_n(\eta) &= 1 + \sum_{m=0}^{r-1} N_{n-m}(\eta)U_m(0) \\ U_n(\eta) &= \eta + \sum_{m=0}^{r-1} M_{n-m}(\eta)U_m(0) \end{aligned}\right\} \tag{11}$$

since $U_m(0) = 0$ for other values of m and

$$0 = 1 + \sum_{m=0}^{r-1} N_{n-m}(0)U_m(0) \qquad (0 \leqq n \leqq r - 1) \tag{12}$$

which expresses the condition of zero load at the breaks in the r filaments identified by $n = 0$ to $n = r - 1$. It should be noted that because of symmetry only the upper half of the sheet $(0 \leqq \eta \leqq 1)$ need be considered. Equations (12) constitute a set of r linear alegebraic equations for the r unknown $U_m(0)$. They can be solved and their solution substituted into equations (11) to yield the entire solution. Before this can be done, however, the influence functions for the particular problem of interest must be determined. Because of the similarities between the influence functions for the two problems, they are determined here simultaneously in order to avoid unnecessary repetition.

Determination of the Influence Functions

The influence functions M_n and N_n are, respectively, the nondimensional displacement and load in the nth filament. These result from displacing the zeroeth filament a unit amount at $\eta = 0$ while maintaining zero displacement at $\eta = 0$ for all other filaments, and also while maintaining at the edge $\eta = 1$ either zero displacement (problem I) or zero load (problem II) in all filaments.

For $\eta \geqq 0$ the problem can be stated as

$$\frac{d^2M_n}{d\eta^2} + k^2\left(M_{n+1} - 2M_n + M_{n-1}\right) = 0 \tag{13}$$

with the conditions

$$
\left.\begin{aligned}
&M_n(0) = 1 && (n = 0) \\
&M_n(0) = 0 && (n \neq 0) \\
&\begin{Bmatrix} M_n(1) \\ \dfrac{dM_n}{d\eta}(1) \end{Bmatrix} = 0 &&
\end{aligned}\right\} \tag{14}
$$

Here and in the remainder of this report the brace symbol $\{\ \}$ means that the upper term corresponds to problem I (uniform normal edge displacement) and the lower term corresponds to problem II (uniform normal edge load).

Let

$$
\overline{M}(\eta, \theta) = \sum_{n=-\infty}^{\infty} M_n(\eta) e^{-in\theta} \tag{15}
$$

or, inversely,

$$
M_n(\eta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \overline{M}(\eta, \theta) e^{in\theta}\, d\theta \tag{16}
$$

Application of equation (15) to equation (13) yields

$$
\frac{\partial^2 \overline{M}}{\partial \eta^2} - 4k^2 \sin^2 \frac{\theta}{2} \overline{M} = 0 \tag{17}
$$

Similar treatment of the boundary conditions in equations (14) yields

$$
\left.\begin{aligned}
&\overline{M}(0, \theta) = 1 \\
&\begin{Bmatrix} \overline{M}(1, \theta) \\ \dfrac{\partial \overline{M}}{\partial \eta}(1, \theta) \end{Bmatrix} = 0
\end{aligned}\right\} \tag{18}
$$

The solutions satisfying equations (17) and (18) are found in

$$
\overline{M}(\eta, \theta) = \cosh\left(2k\eta \left|\sin \frac{\theta}{2}\right|\right) - \begin{Bmatrix} \coth\left(2k \left|\sin \frac{\theta}{2}\right|\right) \\ \tanh\left(2k \left|\sin \frac{\theta}{2}\right|\right) \end{Bmatrix} \sinh\left(2k\eta \left|\sin \frac{\theta}{2}\right|\right)
$$

Substitition of these solutions into equation (16) gives

$$M_n(\eta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\cosh\left(2k\eta \left|\sin\frac{\theta}{2}\right|\right) - \begin{Bmatrix} \coth\left(2k\left|\sin\frac{\theta}{2}\right|\right) \\ \tanh\left(2k\left|\sin\frac{\theta}{2}\right|\right) \end{Bmatrix} \sinh\left(2k\eta\left|\sin\frac{\theta}{2}\right|\right) \right] e^{in\theta}\, d\theta$$

and differentiation yields

$$N_n(\eta) = \frac{k}{\pi} \int_{-\pi}^{\pi} \left[\sinh\left(2k\eta \left|\sin\frac{\theta}{2}\right|\right) - \begin{Bmatrix} \coth\left(2k\left|\sin\frac{\theta}{2}\right|\right) \\ \tanh\left(2k\left|\sin\frac{\theta}{2}\right|\right) \end{Bmatrix} \cosh\left(2k\eta\left|\sin\frac{\theta}{2}\right|\right) \right] \left|\sin\frac{\theta}{2}\right| e^{in\theta}\, d\theta$$

Finally, making an appropriate change of variables and recognizing the symmetry properties of the integrand yield

$$N_n(0) = -\frac{4k}{\pi} \int_0^{\pi/2} \cos 2n\theta \sin\theta \begin{Bmatrix} \coth(2k \sin\theta) \\ \tanh(2k\sin\theta) \end{Bmatrix} d\theta \tag{19}$$

Stress-Concentration Factors

The maximum force occurs at $\eta = 0$ in the first intact filament adjacent to the broken ones. Therefore, for a cut across r filaments the stress-concentration factor K_r is given by $P_r(0)$ which is equal to $P_{-1}(0)$ because of symmetry. Thus,

$$K_r \equiv P_r(0) = 1 + \sum_{m=0}^{r-1} N_{r-m}(0) U_m(0) \tag{20}$$

where values of $N_n(0)$ for problems I and II are obtained from equations (19). The values of $U_m(0)$ are obtained from equations (12) after substitution of the appropriate values for $N_n(0)$.

Closed-form evaluation of the influence functions $N_n(0)$ given by equations (19) does not appear feasible, although asymptotic expressions valid for small values of n and either very small or very large values of k are not too difficult to obtain. Therefore, the influence functions have been evaluated by numerical integration on a digital computer, with the asymptotic expressions being employed in a few cases merely as a check on the numerical results. In addition, the solution of equations (12) and calculation of the stress-concentration factors from equation (20) have been included in the automatic routine.

RESULTS AND DISCUSSION

For both of the problems considered, stress-concentration factors have been calculated for various numbers of broken filaments ranging from 1 to 100 and for a wide range of values of k, the length-stiffness parameter. The results for problem I are presented and discussed first.

Problem I – Uniform Normal Edge Displacement

The stress-concentration factor K_r due to various numbers of broken filaments (r) is plotted in figure 2 as a function of k, the length-stiffness parameter. The results for small numbers of broken filaments are shown in figure 2(a), and for larger numbers in figure 2(b). Also shown are the stress-concentration factors for the infinite-length sheet (see ref. 1) to which the present finite-length results are asymptotic from below for large values of k.

For very small values of k, it is seen in figure 2(a) that the stress-concentration factor is only slightly greater than one and is independent of the number of broken filaments. As k increases, however, the number of broken filaments becomes increasingly important. For example, for $k = 0.5$, no effect of the number of broken filaments is evident; whereas for $k = 10$, different numbers of broken filaments yield significantly different stress-concentration factors, as long as the number of broken filaments is less than about 20. For even larger values of k, the number of broken filaments is of significance over a still wider range. This range is still limited, however, for any finite value of k.

In an experimental program of tensile tests of filamentary composites with cuts, it is likely to be k, the length-stiffness parameter, rather than r, the number of cut filaments, which would be held constant throughout a number of tests. Therefore, it would be informative to present the results in a different way. This is done in figure 3, where the stress-concentration factor is plotted as a function of the number of broken filaments, for various values of k. Although the results are valid only for discrete values of r, they are plotted as continuous curves for ease of illustration. It can be seen that for a given value of k the stress-concentration factor ultimately attains a value which cannot be exceeded, no matter how many filaments are broken. For example, for $k = 4$, the stress-concentration factor has an upper limit of about 2.10; and for $k = 10$, an upper limit of about 3.25. Wherever feasible it would appear to be desirable in many applications to design a composite structural component in such a way that an acceptable upper limit could be imposed on the filament stress-concentration factor. This is analogous to the "fail-safe" philosophy of aircraft design. Within the limitations of the model, this upper limit could be imposed by prescribing a value of k and then suitably adjusting

the relative values of length, matrix shear stiffness, filament extensional stiffness, and filament spacing.

Problem II – Uniform Normal Edge Load

Figure 4(a) contains results for small numbers of broken filaments; the results for larger numbers are presented in figure 4(b). In contrast to the results for problem I, the stress-concentration factors for the uniform-edge-load problem are greater than those for the infinitely long sheet, and they approach them from above as k becomes large. For very small values of k, it can be shown that the stress-concentration factor K_r approaches the value $1 + \frac{r}{2}$. This rather severe stress concentration is largely of academic interest, however, since uniformly loaded edges are rarely encountered in practice. In the event of their occurrence, however, the present results suggest that modifications should be made, perhaps by reinforcing the edges of the sheet, in order that the boundary conditions would correspond more closely to those of problem I.

CONCLUDING REMARKS

Filament stress-concentration factors have been calculated for two problems concerned with the stretching of a finite-length sheet of filamentary material weakened by the presence of a group of broken filaments. For the problem of uniformly loaded edges, the effect of finite length is manifested by an increase in the stress-concentration factor over that for the infinite-length sheet. However, in the problem of uniform edge displacement, the stress-concentration factor is lower for the finite-length sheet than for the infinite-length sheet. In addition, it is seen that to each given value of the length-stiffness parameter there corresponds an upper limit which the stress-concentration factor cannot exceed, regardless of the number of broken filaments. This limit should be of interest to designers working within the constraints of a maximum allowable operating stress.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., July 17, 1970.

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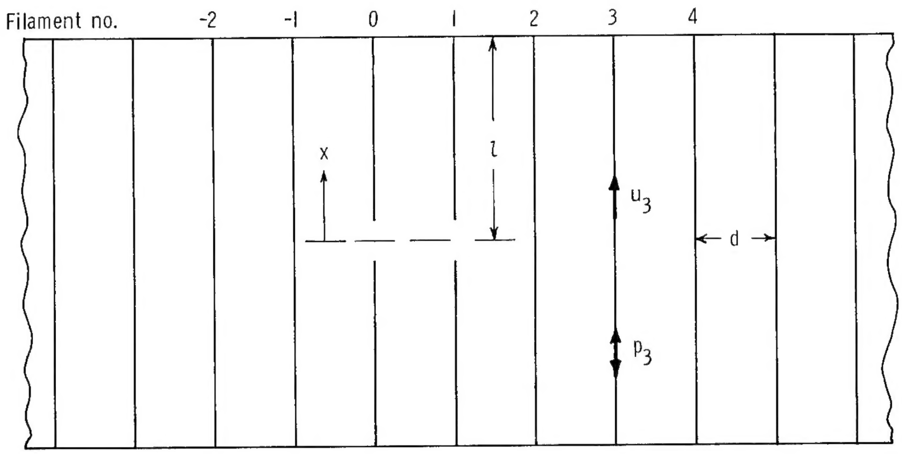
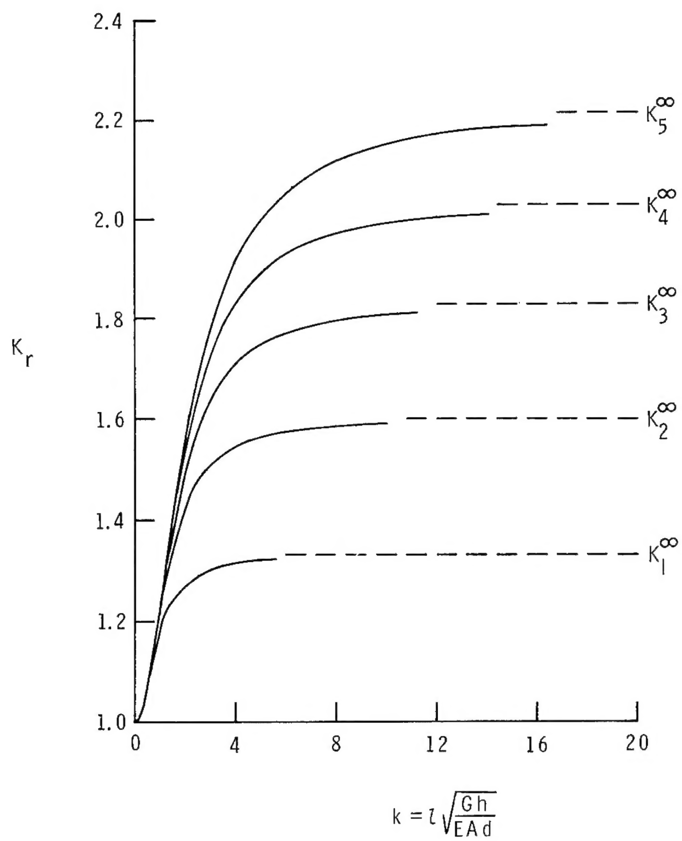
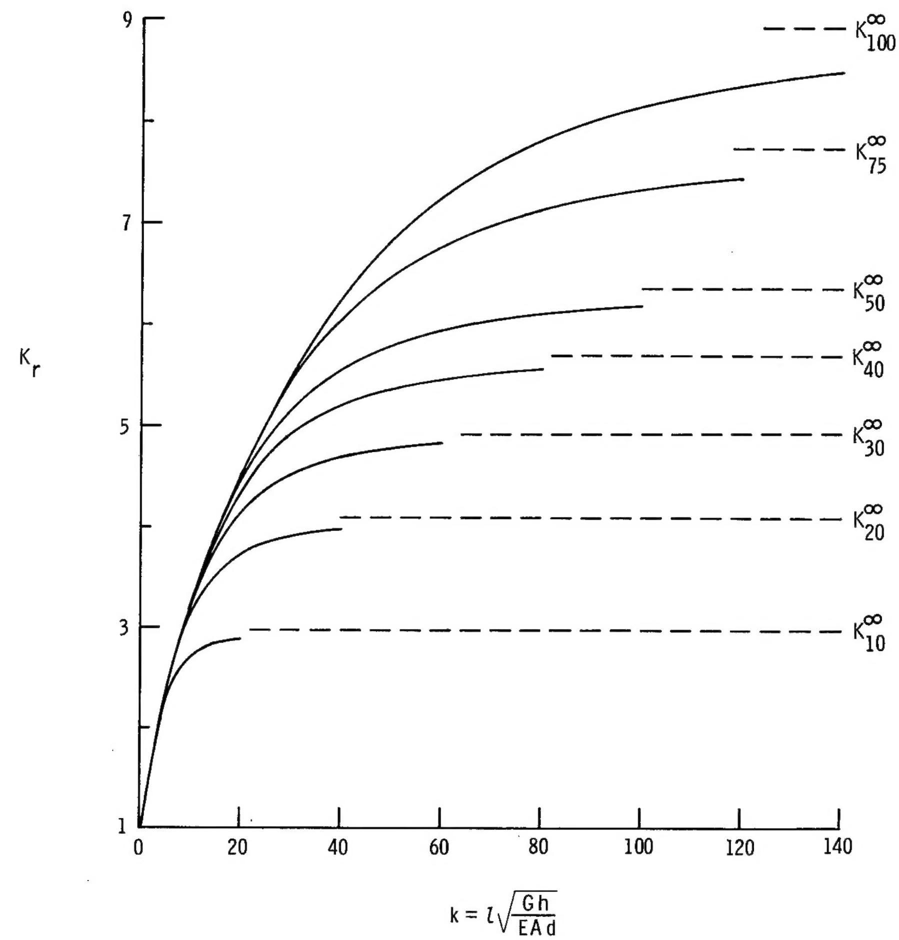


Figure 1.- Coordinate and notation systems for finite-length filament sheet.



(a) One to five broken filaments.

Figure 2.- Stress-concentration factor for r broken filaments as a function of the length-stiffness parameter k. Uniform normal edge displacement.



(b) 10 to 100 broken filaments.

Figure 2.- Concluded.

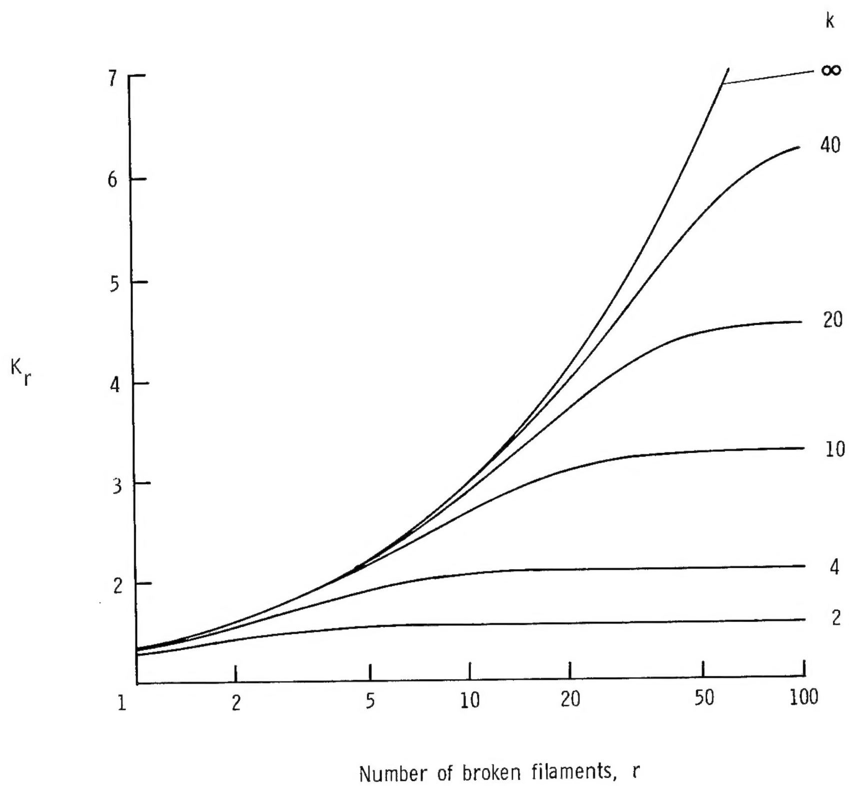
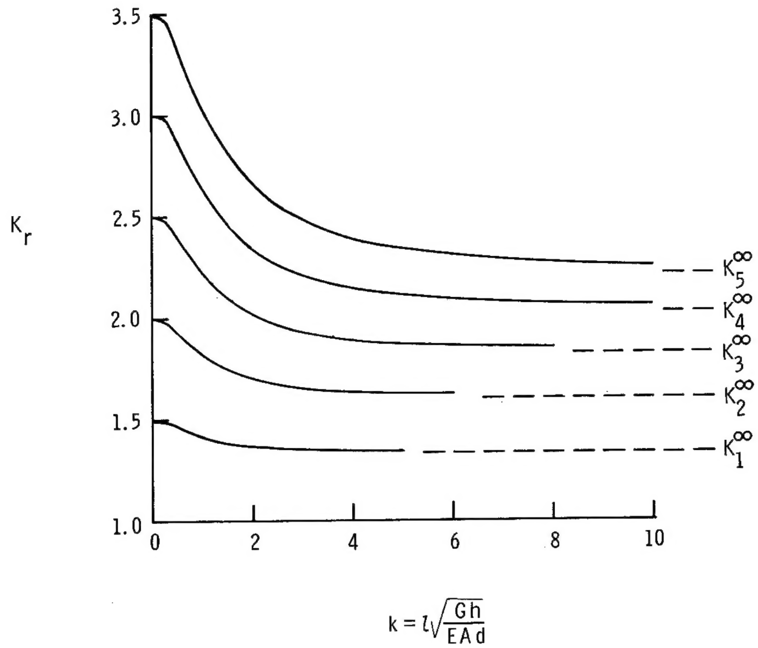
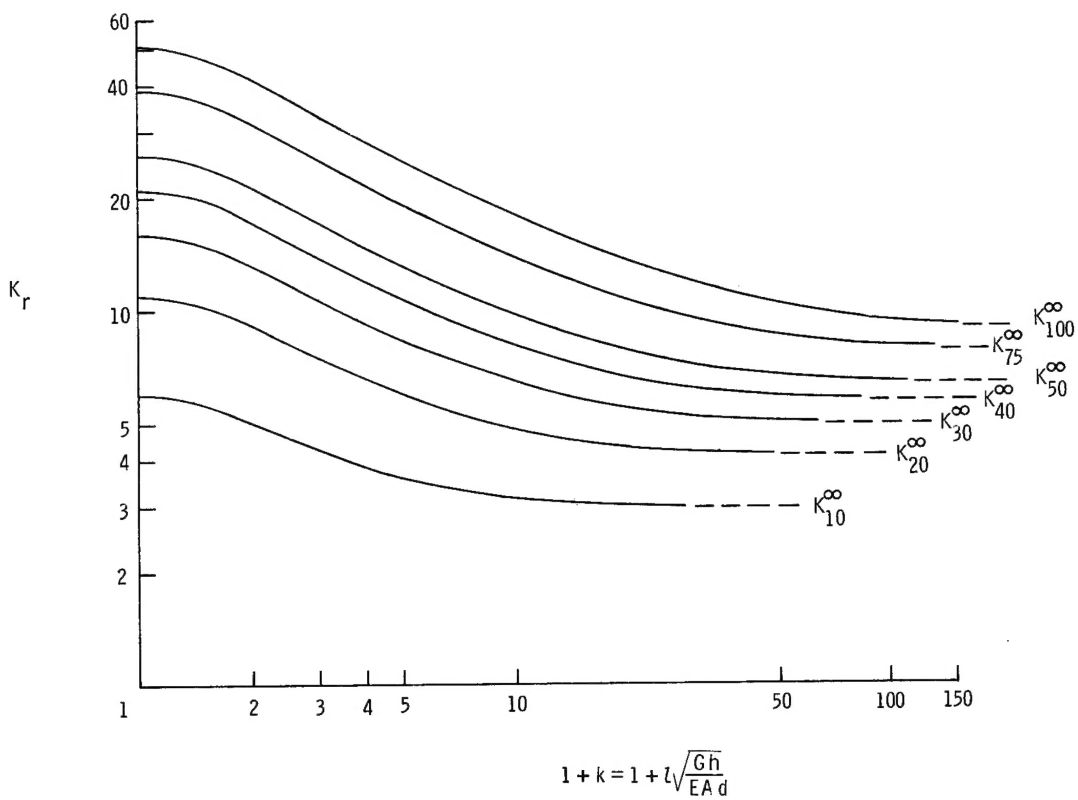


Figure 3.- Stress-concentration factor for fixed values of k as a function of the number of broken filaments. Uniform normal edge displacement.



(a) One to five broken filaments.



(b) 10 to 100 broken filaments.

Figure 4.- Stress-concentration factor for r broken filaments as a function of the length-stiffness parameter k. Uniform normal edge load.